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CONTRIBUTIONS FROM THE OBSERVATORY OF
COLUMBIA UNIVERSITY—No. 27
HAROLD JACOBY, Director

RUTHERFURD PHOTOGRAPHS OF STARS SURROUNDING β CYGNI

BY
MORRIS FRANCIS WEINRICH

SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THE FACULTY OF PURE SCIENCE, COLUMBIA UNIVERSITY

PUBLISHED FROM THE
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RUTHERFURD PHOTOGRAPHS OF STARS SURROUNDING β CYGNI.

Introductory. — The present paper contains results derived from fourteen negatives made by Rutherfurd in the years 1875 and 1876. The plates obtained in 1875 were measured in the same year under Rutherfurd's direction and were reduced in 1892,[1] while those taken in 1876 were measured at Columbia University in the spring of 1899, and are here discussed for the first time.

All the plates under consideration were made with Rutherfurd's 13-inch objective and photographic corrector. As usual, three exposures were impressed on each plate, the first two having a duration of about six minutes and the last being just long enough to obtain a third image of β and β_2, the two brightest stars. In making these exposures the clockwork was alternately thrown in and out of action so that the distance on the plates between the first and second images is about 1.0 mm. in right ascension while the third images are about 37.5 mm. to the west of the first. The object of this third exposure was to afford means for determining the orientation correction of the plate. We have used the third image of the central star only for the adjustment of the plate in the measuring machine, the orientation correction being determined from known stars.

The telescope and plate holder were so adjusted by Rutherfurd that the double star β Cygni coincided approximately with the plate centre, defined as the point where the optical axis of the lenses normally pierces the plate.

Table I gives the data of exposure; excepting the "zenith distance" which was computed by the usual formulæ, all were inscribed upon the plate by Rutherfurd himself. The "sidereal time" is the mean of the four instants of beginning and

[1] The Rutherfurd photographic measures of the stars about β Cygni. By Harold Jacoby. *Annals N. Y. Acad. Sci.*, VI, May, 1892.

ending of the two six-minute exposures. Of the three thermometers used, the first was attached to the mercurial barometer, the second was in the open air, and the third fastened to the telescope tube. The column "focus" contains readings of a micrometer on the plate end of the telescope indicating on an arbitrary scale the distance of the plate holder from the objective and intended by Rutherfurd to be used in the determination of the scale value. These readings were not used in the reductions as the scale value was separately determined for each plate from the position of known stars.

TABLE I.—PHOTOGRAPHS OF β CYGNI.

OBSERVATORY OF L. M. RUTHERFURD, NEW YORK.

Lat. 40° 43′ 48″.5. Long. 4^h 55^m $56^s.62$ W.

Plate No.	Date.	Sidereal Time.	Barometer.	Thermometer.			Focus.	Zenith Dist.
				Att.	Ext.	Tel.		
		h m s	in.					°
1	1875 July 26	18 45 52	30.10	75°	74°	76°	7.6	15.40
2	1875 July 26	19 13 28	30.10	75	74	76	7.6	13.28
3	1875 July 26	19 38 42	30.10	75	74	76	7.6	13.32
4	1875 Sep. 20	20 27 22	29.95	57	55	60	7.7	18.18
5	1875 Sep. 20	21 53 52	29.95	57	55	60	7.9	32.98
6	1875 Sep. 20	22 20 12	29.95	57	55	60	7.7	37.90
7	1876 May 31	16 10 48	30.26	56	53	60	7.7	41.76
8	1876 May 31	17 0 38	30.26	56	53	60	7.7	32.40
9	1876 June 1	15 25 48	29.95	60	58	60	7.7	50.28
10	1876 June 1	15 53 38	29.95	60	58	60	7.7	45.00
12	1876 Oct. 24	22 38 2	29.69	59	55	60	7.7	41.30
13	1876 Oct. 24	23 9 32	29.69	59	55	60	7.7	47.26
14	1876 Nov. 4	21 45 18	30.29	52	50	55	7.75	31.40
15	1876 Nov. 4	22 33 2	30.29	52	50	55	7.75	40.30

A plate of the series marked No. 11 has never been found among the Rutherfurd photographs now owned by Columbia University and was perhaps an unsuccessful exposure made on October 24, 1876.

Statement of Problem. — The fourteen plates above listed naturally fall into two "Series" and four "Sets" in the following manner:

Series (A).		Series (B)	
Set (1). Plate.	Set (2). Plate.	Set (3). Plate.	Set (4). Plate.
1	4	7	12
2	5	8	13
3	6	9	14
		10	15

As aforesaid the plates of Sets (1) and (2) were completely reduced[1] in 1892. In the course of reduction a curious displacement of β Cygni was noted when the corrected Mean Distances and Position angles of Sets (1) and (2) were compared. Jacoby considered the displacement, which varied with position angle only, as possibly parallactic and deduced a value of $+ 0''.97$[2] for the parallax. Later after introducing data obtained from plates of Sets (3) and (4) a smaller value amounting to about $+ 0''.61$[3] was found. Hereupon Chase[4] made a heliometer search for parallax of this star and found a value of $- 0''.018 \pm 0''.016$. This heliometric result seemed to indicate with certainty that the photographic displacement was not due to parallax, and thereby left the real cause undetermined.

It was therefore decided to so plan the work as to obtain four separate positions of β Cygni corresponding to the mean epochs of Sets (1), (2), (3) and (4). It was hoped that from these results positive conclusions could be drawn. This formed one part of the problem. The other part was the determination of the positions of 43 stars about β Cygni. This part of the work was divided into four steps as follows:

First: An intercomparison of the separate plates of Sets (3) and (4).

Second: A reduction of two sets of mean results derived from Sets (3) and (4), using the *Astronomische Gesellschaft* Catalog as standard.

Third: A reduction of two sets of mean results derived from the mean coördinates of Sets (1) and (2) using the mean of the two positions of the stars as obtained in step two as standard.

[1] *Annals N. Y. Acad. Sci.*, VI, May, 1892.
[2] *A. J.*, Vol. XII, No. 287, p. 182.
[3] *A. J.*, Vol. XIII, No. 293, p. 37.
[4] *A. J.*, Vol. XIII, No. 307, p. 162.

Fourth: A reduction of final position from the mean results of steps two and three and a calculation of the precession constants and the magnitudes.

Measurement of Plates. — The plates of Series (B) were measured in the spring of 1899 by Miss F. E. Harpham, assisted by Miss Tarbox, in the usual Repsold machine. This machine corresponds very closely to the description given by Scheiner[1] of a similar instrument in use at the Leyden Observatory, differing only in that an alteration has been made which obviates "projection errors." This alteration consists of a contrivance by means of which it is possible to rotate the slide of the microscope through a small angle with great accuracy and without jar, so that the same micrometer is used for bisecting the star and setting on the scale.

The reading microscope is provided with both horizontal and vertical screws but in these measures only the horizontal screw was used. The screw has a pitch of one half millimeter, two revolutions corresponding to one millimeter scale division. The micrometer head is divided into one hundred parts, so that by estimating tenths of a division the scale may be read to half-microns (0.0005 mm.) which corresponds to about $0''.026$ on the Rutherford plates.

Each plate was measured in four positions, designated as "x direct," "y direct," "x reversed" and "y reversed." Both images of each star were observed in the order: first image direct, second image direct, first image reversed, second image reversed, in the x and y direction, giving four separate x and y readings each of which was itself determined from four separate and distinct settings on both the star and the scale.

Instrumental corrections were applied to these measures as follows:

1. Screw errors of the microscope.
2. Corrections for runs.
3. Division errors of the measuring machine scale.
4. "Rotation corrections" to render the two coördinates perfectly rectangular.

[1] Die Photographie der Gestirne, p. 148.

After the corrections had been applied to the four separate x readings they were reduced to the center by comparing the first and second with the direct and the third and fourth with the reversed reading for the central star, the average of these four results now giving the x-coördinate of the star fully corrected for all instrumental errors as given in Table II. The y-coördinates given in the table were obtained in an exactly similar manner.

Reduction of Coördinates.— The first step in the reduction of the coördinates given in Table II is to change them from millimeters to seconds of arc approximately, using a scale value 1 mm. = 52″.87 which is an average for Rutherfurd plates. A small correction to this scale value will be determined as one of the constants for each plate.

Refraction.— Refraction affects both position angles and distances of stars upon the photographic plate. Of the various formulæ which have been developed for the determination of its effect upon the x- and y-coördinates we have used those of Jacoby[1] first deduced by him from formulas published by Rambaut.[2]

Let:

φ = the latitude of Rutherfurd's Observatory.

θ = the sidereal time of exposure as given in Table I.

α and δ = the right ascension and declination of the plate-centre approximately.

β = the usual refraction constants increased by 1/65 to allow for the difference between photographic and visual refractions.

If we then compute the auxiliary quantities:

$$\begin{aligned}
\tan N &= \cot \phi \cos (\theta - \alpha), \\
G &= \cot (\delta + N), \\
H &= \tan (\theta - \alpha) \sin N \operatorname{cosec} (\delta + N), \\
M_x &= \beta(1 + H^2) \sin 1'', \\
N_x &= \beta(GH - H \tan \delta) \sin 1'', \\
M_y &= \beta(GH + H \tan \delta) \sin 1'', \\
N_y &= \beta(1 + G^2) \sin 1''.
\end{aligned}$$

[1] *A. J.*, Vol. XVII, No. 387, p. 22.

[2] *A. N.*, Band 131, No. 3125, p. 65.

TABLE II. — COÖRDINATES CORRECTED FOR INSTRUMENTAL ERRORS.

Star.	PLATE 7.		PLATE 8.		PLATE 9.		PLATE 10.	
	X	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*
2			−55.4254	+ 0.0821			−55.4140	+ 0.0748
3	−43.5142	+31.2929	−43.5139	+31.3079	−43.4983	+31.3008	−34.5099	+31.3105
5			−38.9484	+54.3317	−38.9267	+54.3283		
6	−37.8604	− 2.3985	−37.8647	− 2.3883	−37.8550	− 2.3899	−37.8586	− 2.3932
8	−34.1682	−40.3442	−34.1700	−40.3340	−34.1819	−40.3318	−34.1689	−40.3405
9	−32.2997	+35.9055	−32.2979	+35.9174	−32.2782	+35.9088	−32.2900	+ 35.9114
11	−24.8720	− 2.0837	−24.8756	− 2.0776	−24.8667	− 2.0787	−24.8697	− 2.0842
12	−21.2346	−24.5238	−21.2338	−24.5174	−21.2313	−24.5180	−21.2365	−24.5200
13	−20.7890	+ 7.2121	−20.7867	+ 7.2185	−20.7795	+ 7.2194	−20.7804	− 7.2186
14	−15.7834	−40.6728	−15.7894	−40.6630	−15.8010	−40.6640	−15.7908	−40.6652
15	−11.7102	+51.8598	−11.7108	+51.8703	−11.6861	+51.8580	−11.7046	+51.8732
16	− 3.8466	−49.2306	− 3.8491	−49.2314	− 3.8612	−49.2262	−- 3.8580	−49.2312
17	− 2.6454	+21.5084	− 2.6438	+21.5118	− 2.6311	+21.5077	− 2.6417	+21.5093
18	− 1.9040	+25.4793	− 1.8992	+25.4825	− 1.8845	+25.4803	− 1.8994	+25.4822
19	− 0.5292	− 0.3746	− 0.5336	− 0.3745	− 0.5278	− 0.3722	− 0.5303	− 0.3762
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21	+ 1.3889	−25.2724	+ 1.3888	−25.2680	+ 1.3832	−25.2714	+ 1.3908	−25.2711
22	+ 1.6803	+39.1310	+ 1.6806	+39.1342	+ 1.6892	+39.1273	+ 1.6872	+39.1337
23	+ 5.7187	−42.8041	+ 5.7162	−42.8029	+ 5.7078	−42.8027	+ 5.7097	−42.8048
24	+ 6.5700	− 1.0076	+ 6.5732	− 1.0066			+ 6.5852	− 1.0022
25	+ 7.7538	+34.8777	+ 7.7600	+34.8805	+ 7.7792	+34.8763	+ 7.7736	+34.8813
26	+11.6805	+50.9916	+11.6854	+50.9958	+11.7022	+50.9892	+11.6928	+50.9936
27	+20.3438	+20.2870	+20.3459	+20.2868	+20.3496	+20.2809	+20.3472	+20.2850
28	+20.3451	+20.3948	+20.3519	+20.3911	+20.3580	+20.3846	+20.3566	+20.3900
30	+24.6524	− 3.1916	+24.6511	− 3.1953	+24.6477	− 3.1982	+24.6537	− 3.1940
31	+25.9586	−37.1298	+25.9612	−37.1300	+25.9430	−37.1356	+25.9573	−37.1256
32	+35.2589	−22.1788	+35.2567	−22.1810			+35.2541	−22.1816
35	+36.3166	−39.6183	+36.3158	−39.6168	+36.3018	−39.6268	+36.3106	−39.6218
36	+36.3274	+37.6380	+36.3320	+37.6342	+36.3396	+37.6319	+36.3376	+37.6343
37	+37.7769	−28.9343	+37.7781	−28.9364			+37.7688	−28.9350
38	+38.2992	−15.7404	+38.3049	−15.7481	+38.2915	−15.7526	+38.2979	−15.7465
39	+41.9841	+34.8416	+41.9993	+34.8349			+42.0071	+34.8400
39½	+43.3686	+26.4112	+43.3723	+26.4030			+43.3619	+26.4028
40	+47.8880	+21.4435	+47.8967	+21.4363			+47.8925	+21.4398
41	+50.4083	+14.3176	+50.4248	+14.3079			+50.4226	+14.3175
41½	+51.5831	+45.2704	+51.5905	+45.2768	+51.5933	+45.2702		
42	+51.7632	+30.1683	+51.7684	+30.1617	+51.7760	+30.1579	+51.7694	+30.1668

TABLE II.—COÖRDINATES CORRECTED FOR INSTRUMENTAL ERRORS.—*Concluded.*

Star.	PLATE 12.		PLATE 13.		PLATE 14.		PLATE 15.	
	X	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*
2			−55.4005	+ 0.0526	−55.4162	+ 0.0523	−55.4112	+ 0.0572
3	−43.5252	+31.3016	−43.5316	+31.2859	−43.5364	+31.2919	−43.5264	+31.2854
5							−38.9732	+54.3287
6	−37.8464	− 2.3886	−37.8363	− 2.4070	−37.8561	− 2.4069	−37.8478	− 2.4011
8	−34.1390	−40.3443	−34.1267	−40.3437	−34.1317	−40.3519	−34.1329	−40.3476
9	−32.2948	+35.9230	−32.3150	+35.9084	−32.3180	+35.9071	−32.3139	+35.9076
11	−24.8556	− 2.0755	−24.8600	− 2.0835	−24.8662	− 2.0864	−24.8584	− 2.0832
12	−21.2110	−24.5216	−21.2020	−24.5274	−21.2112	−24.5300	−21.2021	24.5204
13					−20.7869	+ 7.2090	−20.7823	+ 7.2118
14	−15.7560	−40.6654	−15.7398	−40.6644	−15.7529	−40.6757	−15.7446	−40.6697
15	−11.7214	+51.8875	−11.7533	+51.8663			−11.7426	+51.8822
16	− 3.8190	−49.2224	− 3.7975	−49.2302	− 3.8038	−49.2294	− 3.8036	−49.2294
17	− 2.6396	+21.5182	− 2.6466	+21.5102	− 2.6532	+21.5167	− 2.6472	+21.5192
18	− 1.8981	+25.4899	− 1.8994	+25.4914	− 1.9102	+25.4891	− 1.9155	+25.4938
19	− 0.5363	− 0.3736	− 0.5357	− 0.3731	− 0.5451	− 0.3714	− 0.5438	− 0.3692
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
21					+ 1.4157	−25.2628		
22			+ 1.6462	+39.1330	+ 1.6588	+39.1459	+ 1.6575	+39.1472
23	+ 5.7538	−42.8028	+ 5.7622	−42.7933	+ 5.7604	−42.7973	+ 5.7601	−42.7969
24					+ 6.5955	− 0.9886		
25	+ 7.7698	+34.8877	+ 7.7482	+34.8801	+ 7.7444	+34.8944	+ 7.7446	+34.8952
26	+11.6764	+51.0075	+11.6526	+51.0046	+11.6548	+51.0169	+11.6592	+51.0135
27	+20.3544	+20.2944	+20.3376	+20.2958	+20.3421	+20.3052	+20.3445	+20.3018
28								
30	+24.6758	− 3.1816	+24.6663	− 3.1781	+24.6659	− 3.1754	+24.6661	− 3.1769
31	+25.9831	−37.1230	+25.9950	−37.1134	+25.9985	−37.1019	+25.9958	−37.1094
32								
35	+36.3458	−39.6200	+36.3622	−39.6046	+36.3620	−39.5969	+36.3616	−39.6068
36	+36.3340	+37.6473			+36.3200	+37.6653	+36.3230	+37.6648
37	+37.8067	−28.9376			+37.8156	−28.9147	+37.8190	−28.9225
38	+38.3270	− 15.7388	+38.3278	−15.7303	+38.3287	−15.7203	+38.3341	−15.7250
39								
39½								
40								
41								
41½								
42	+51.7734	+30.1804	+51.7510	+30.1889	+51.7671	+30.2057	+51.7640	+30.2000

we can form the equations :

$$X = X' + M_x X' + N_x Y'$$
$$Y = Y' + M_y X' + N_y Y'.$$

in which X' and Y' denote the coördinates as given in Table II, and X and Y the same coördinate corrected for refraction.

The refraction coefficients obtained by the above formulæ for the plates of Series (B) are given in the subjoined table.

TABLE III.—REFRACTION COEFFICIENTS.

Plate.	M_x	N_x	M_y	N_y
7	+0.000455	+0.000014	—0.000217	+0.000349
8	+ .000368	+ .000027	— .000133	+ .000322
9	+ .000578	— .000030	— .000333	+ .009392
10	+ .000488	+ .000002	— .000251	+ .000357
12	+ .000439	— .000015	+ .000207	+ .000339
13	+ .000521	+ .000009	+ .000282	+ .000369
14	+ .000364	— .000027	+ .000127	+ .000323
15	+ .000442	— .000017	+ .000203	+ .000346

Twenty-one stars which occurred on all the plates of Series (B), and termed "Standard Stars" were by means of these constants corrected for refraction and their X- and Y-coördinates found.

Precession, Nutation and Aberration. — Precession and nutation merely rotate the axes of reference, not affecting in any way the position of the stars relative to each other. The effect of aberration is to alter the distance between the stars by a constant factor and to change the position angles equally in all directions. No corrections were therefore applied for precession, nutation and aberration as the rotation effects can be taken care of by the orientation constant and the alteration of the distances by the correction to the scale value.

Plate Constants. — The next step was an intercomparison of the plates in Set (3) and (4) taken separately. For this purpose the X- and Y-coördinates of the twenty-one "Standard Stars" were combined into arithmetical means designated by x_3, y_3, and x_4, y_4. In order to avoid confusion in carrying the two sets along together, let us devote our attention to Set (3). For this we now have twenty-one pairs of mean coördinates belonging to an

imaginary mean plate. We now wish to obtain plate constants for each plate which will bring about the best possible agreement of that plate with the imaginary mean plate. These constants are obtained for the purpose of reducing the non-standard stars to the imaginary mean plate. For this purpose, we let:

p = a correction required by the assumed scale value.
r = a small angle through which the coördinate axes of the plate must be rotated, called the orientation correction.
k, c = corrections of the plate-center in x and y.

Then there results for each of the twenty-one stars a pair of equations of the form

$$pX + rY + k = x_3 - X$$
$$pY - rX + c = y_3 - Y$$

where the right-hand members are the coördinate differences between the mean plate and each individual plate of the set, taken in the sense mean *minus* plate.

These equations would be rigorously true and the four plate constants could be determined from any pair of known stars if it were not for the unavoidable accidental errors of measurement. On account of these errors it is of great advantage to have as many equations as possible and to solve for the plate constants by the method of least squares.

Owing to the peculiar arrangement of the unknowns in these equations we do not need to make the least square solution in the usual manner but may use the following auxiliary quantities as pointed out by Jacoby.[1] In these let ν be the number of comparison stars, and let us denote by square brackets the sum of ν quantities.

Put:

$$A = [XX] - \frac{[X]^2}{\nu},$$

$$C = [X(X-x)] - \frac{[X][X-x]}{\nu},$$

[1] H. Jacoby. Note on the Solution by Least Squares of the Equations arising in the Reduction of Photographic Star-plates. *Mo. Not. R.A.S.*, May, 1896, p. 424.

$$D=[YY]-\frac{[Y]^2}{\nu},$$

$$E=[Y(X-x)]-\frac{[Y][X-x]}{\nu},$$

$$C'=[Y(Y-y)]-\frac{[Y]\,Y-y]}{\nu},$$

$$E'=-[X(Y-y)]+\frac{[X][Y-y]}{\nu}.$$

Then :

$$p=-\frac{C+C'}{A+D},\quad \text{weight of } p=A+D,$$

$$r=-\frac{E+E'}{A+D},\quad \text{weight of } r=A+D,$$

$$k=-\frac{1}{\nu}\{[X]p+[Y]r+[X-x]\},\quad \text{weight of } k=\nu-\frac{[X]^2+[Y]^2}{[XX]+[YY]},$$

$$c=-\frac{1}{\nu}\{[Y]p-[X]r+[Y-y]\},\quad \text{weight of } c=\nu-\frac{[X]^2+[Y]^2}{[XX]+[YY]}.$$

If $[vv]$ represents the sum of the squares of the residuals of the ν right ascension equations plus the ν declination equations the probable error of a single equation will will be

$$\pm 0.6745\sqrt{\frac{[vv]}{2\nu-4}}$$

and the probable error of one of the unknowns will be this quantity divided by the square root of the weight of the unknown, as given above.

A very convenient check on the computations is given by the fact that the sum of the residuals of the right ascension, as well as of the declination equations must be approximately equal to zero.

Using the above method in solving the forty-two equations for each plate of Set (3) we obtain values for p, r, k and c, suitable for the reduction of each individual plate to the mean plate, these together with the values obtained for Set (4), are contained in the following table.

TABLE IV.—PLATE CONSTANTS.

Plate.	p	r	Probable Error of p or r.	k	c	Probable Error of k or c.	Probable Error of One Equation.
				"	"	"	"
7	+0.000004	+0.000130	±0.000012	+0.027	+0.071	±0.026	±0.1207
8	+ .000024	+ .000066	.000006	+ .046	— .100	.014	.0621
9	— .000008	— .000207	.000009	— .102	+ .056	.019	.0880
10	— .000027	— .000017	.000009	— .018	— .030	.019	.0852
12	— .000016	— .000270	.000011	— .137	+ .022	.022	.1027
13	+ .000045	+ .000098	.000011	— .030	+ .109	.024	.1109
14	— .000017	+ .000101	.000009	+ .154	— .072	.020	.0898
15	— .000013	+ .000072	.000010	+ .014	— .061	.021	.0962

The coördinates of the non-standard stars as given in Table II after having been reduced to seconds of arc were now corrected as follows:

$$\text{for } X'', \quad (p + M_x)X'' + (r + N_x)Y'' + k,$$
$$\text{for } Y'', \quad (-r + M_y)X'' + (p + N_y)Y'' + c.$$

The means of the corrected coördinates of the non-standard stars were now taken separately for the two sets, and these, together with the means of the standard stars corrected for refraction, gave us mean coördinates of all stars for each set. It was, of course, not necessary to apply the corrections, as given in the last two equations, to the standard-stars as these have already been corrected for refraction, and the correction

$$\text{for } X'', \quad pX'' + rY'' + k,$$
$$\text{for } Y'', \quad pY'' - rX'' + c,$$

would not have appreciably altered the mean coördinates already formed on account of the method by which p, r, k and c were obtained.

Having thus reduced all coördinates to two mean systems, Tables V and VI were formed. These tables give the arithmetrical averages of all coördinates reduced as above, together with the differences between each plate and the average, taken in the sense plate *minus* average.

Standard Coördinates of Series (B). — The mean coördinates of Tables V and VI were next corrected in such a way as to

TABLE V. — MEAN X'S AND Y'S AND RESIDUALS FOR EACH PLATE OF SET (3).

Star.	x_3	7	8	9	10	y_3	7	8	9	10
	"	"	"	"	"	"	"	"	"	"
2	−2931.28		−.16		+.17	+ 4.72		+.10		−.10
3	−2301.41	+.02	+.12	−.16	−.07	+1656.12	−.20	+.06	−.26	+.33
5	−2059.84		+.15	−.16		+2873.79		−.03	+.03	
6	−2002.59	.00	−.07	−.02	+.07	− 126.07	− .02	+.06	−.02	−.07
8	−1807.54	−.04	+.11	−.29	+.22	−2132.99	−.06	+.05	+.11	−.12
9	−1708.05	−.12	+.01	−.03	+.04	+1899.68	−.03	+.17	−.18	−.04
11	−1315.55	−.02	−.09	+.02	+.07	− 109.76	+.08	+.03	+.04	−.19
12	− 1123.18	−.17	+.04	+.25	−.11	−1296.56	−.01	.00	−.02	.00
13	−1099.37	−.16	+.04	−.06	+.17	+ 381.96	−.08	−.07	+.08	+.05
14	− 835.28	+.14	.00	−.19	+.05	−2150.59	−.18	+.06	+.02	+.07
15	− 619.04	+.06	−.04	+.10	−.13	+2743.23	−.15	+.12	−.31	+.33
16	− 203.85	+.04	+.08	+.11	−.20	−2603.66	+.06	−.16	+.16	−.03
17	− 139.66	−.06	−.02	+.10	−.12	+1137.64	+.03	+.02	−.01	−.05
18	− 100.32	−.17	+.04	+.21	−.19	+1347.68	−.01	−.04	+.04	.00
19	− 28.04	+.07	−.14	+.02	−.03	− 19.80	+.06	−.11	+.17	−.12
20	0.00	+.03	+.05	−.10	−.02	0.00	+.07	−.10	+.06	−.03
21	+ 73.41	−.11	−.02	−.02	+.15	−1336.55	−.03	+.06	−.01	−.02
22	+ 89.07	+.14	+.05	−.30	+.12	+2069.60	+.03	+.03	−.10	+.03
23	+ 302.19	.00	−.02	+.18	−.16	−2263.90	+.01	−.03	+.07	−.04
24	+ 347.85	−.31	−.13		+.45	− 53.29	−.05	−.12		+.17
25	+ 410.82	−.39	−.17	+.16	+.31	+1844.60	−.04	−.01	+.02	+.06
26	+ 618.37	+.88	−.02	−.06	+.06	+2696.79	−.05	+.07	+.02	−.05
27	+1076.24	+.02	+.02	−.10	−.03	+1072.59	+.05	+.03	−.01	−.05
28	+1076.55	−.22	+.02	+.03	+.16	+1078.16	+.19	−.02	−.10	−.06
30	+1303.92	+.05	−.07	−.13	+.11	− 169.27	+.09	−.08	+.01	+.01
31	+1372.88	−.08	+.10	−.13	+.13	−1964.09	−.06	−.02	−.14	+.27
32	+1864.76	+.10	−.07		−.02	−1173.59	+.01	.00		−.01
35	+1920.68	−.02	−.06	+.09	−.03	−2095.95	−.01	+.20	−.11	−.02
36	+1921.88	−.05	−.02	−.08	+.13	+1990.01	+.01	−.08	+.17	−.10
37	+1997.92	+.07	+.10		−.16	−1530.87	−.05	−.03		+.08
38	+2025.79	−.07	+.16	−.08	−.05	− 833.31	+.19	−.08	−.05	+.01
39	+2221.50	−.48	+.09		+.39	+1841.97	+.06	−.15		+.08
39½	+2293.97	+.21	+.20		−.41	+1395.93	+.19	−.08		−.12
40	+2533.28	−.09	+.17		−.07	+1133.24	+.07	−.11		+.03
41	+2666.87	−.42	+.25		+.17	+ 756.35	+.04	−.26		+.22
41½	+2728.79	+.03	+.14	−.17		+2393.82	−.41	+.15	+.26	
42	+2738.34	−.11	−.07	+.14	−.07	+1594.68	+.01	−.12	+.10	+.10

TABLE VI.—MEAN X'S AND Y'S AND RESIDUALS FOR EACH PLATE OF SET (4).

Star.	x_4	12	13	14	15	y_4	12	13	14	15
	"	"	"	"	"	"	"	"	"	"
2	—2930.76		+.05	+.04	—.08	+ 2.52		—.17	+.10	+.06
3	—2302.46	—.30	—.22	+.17	+.33	+1654.46	—.09	.00	+.31	—.22
5	—2061.22				.00	+2872.99				.00
6	—2001.84	—.05	+.26	—.17	—.02	— 127.39	+.13	—.18	—.02	+.08
8	—1805.36	+.14	—.20	+.19	—.13	—2134.24	—.28	+.16	+.07	+.05
9	—1709.03	+.20	—.25	+.08	—.03	+1898.94	+.13	+.12	—.07	—.16
11	—1314.93	+.14	—.20	—.06	+.11	— 110.39	+.02	+.06	—.07	—.02
12	—1121.67	.00	—.08	—.10	+.16	—1297.30	—.09	—.10	—.10	+.27
13	—1099.20			.00	.00	+ 381.19			—.03	+.03
14	— 832.96	+.06	+.07	—.19	+.08	—2151.07	+.02	+.21	—.20	—.02
15	— 621.11	+.22	—.38		+.17	+2743.66	+.24	—.35		+.12
16	— 201.28	—.11	+.10	+.06	—.04	—2603.62	+.32	—.19	—.02	—.09
17	— 140.00	—.08	+.09	—.08	+.06	+1137.92	+.08	—.13	—.06	+.10
18	— 100.82	—.09	+.46	+.04	—.40	+1348.16	—.10	+.22	—.20	+.08
19	— 28.57	+.08	+.21	—.11	—.18	— 19.68	—.08	+.04	—.04	+.08
20	0.00	- .14	—.03	+.15	+.01	0.00	+.02	+.11	—.07	—.06
21	+ 74.93			.00		—1336.12			.00	
22	+ 87.70		—.43	+.33	+.09	+2070.17		—.22	+.03	+.18
23	+ 304.64	+.19	—.09	.00	—.08	—2263.42	—.13	+.18	—.02	—.02
24	+ 348.97			.00		— 52.35			.00	
25	+ 410.00	+.30	+.05	—.12	—.25	+1845.32	+.01	—.26	+.06	+.20
26	+ 616.74	—.06	—.07	+.02	+.10	+2697.98	—.02	—.03	+.05	+.02
27	+1076.08	+.07	—.14	+.01	+.07	+1073.82	+.02	—.03	+.01	—.02
28										
30	+1304.80	+.27	.00	—.12	—.13	— 167.81	+.19	+.06	—.15	—.10
31	+1374.89	—.16	+.01	+.13	.00	—1962.50	—.16	—.13	+.28	.00
32										
35	+1923.12	—.25	+.18	+.01	+.05	—2094.36	—.09	+.06	+.20	—.15
36	+1921.21	—.13		.00	+.13	+1992.04	—.05		—.09	+.15
37	+2000.10	—.11		—.06	+.18	—1529.37	—.08		+.16	—.07
38	+2027.38	—.08	+.04	—.14	+.18	— 831.44	+.05	—.09	+.04	—.02
39										
39½										
40										
41										
41½										
42	+2737.95	—.12	—.18	+.21	+.08	+1597.46	+.03	—.10	+.02	+.04

bring about the best possible agreement with the star places in the *Astronomische Gesellschaft* Catalog. Table VII shows which stars of Series (B) appear in the *Astronomische Gesellschaft* Catalog.

TABLE VII.—CONCORDANCE WITH ASTRONOMISCHE GESELLSCHAFT CATALOG.

Photo. No. of Star.	No. in A. G. Catalog.	Photo. No. of Star.	No. in A. G. Catalog.	Photo. No. of Star.	No. in A. G. Catalog.	Photo. No. of Star.	No. in A. G. Catalog.
2	9989	14	10036	23	10056	36	10090
3	10006	15	10039	24	10059	37	10093
5	10009	16	10047	25	10061	38	10094
6	10010	17	10048	26	10066	39½	10098
8	10019	19	10049	27	10074	40	10104
9	10020	20	10052	30	10079	41½	10110
11	10026	21	10053	31	10081	42	10111
12	10032	22	10054	35	10089		

NOTE. — No. 19 is β Cygni and No. 20 is β_2 Cygni.

The method used to procure this agreement was that published by Jacoby.[1] In this method, rectangular coördinates x_c and y_c are computed from the *Astronomische Gesellschaft* Catalog places, and then a series of equations similar to that used before are solved. These equations are:

For Set (3)

$$px_3 + ry_3 + k + (x_3 - x_c) = 0,$$
$$py_3 - rx_3 + c + (y_3 - y_c) = 0,$$

those for Set (4) having the mean values of the coördinates for that set substituted for x_3 and y_3. The rectangular coördinates computed from the *Astronomische Gesellschaft* Catalog are here designated by x_c and y_c. These were computed by the following formulæ as given in the paper just mentioned. Terms of the fourth order were inappreciable.

$$x_c = x' + J_1'x'y', \qquad J_1' = -\tan\delta \sin 1'',$$
$$+ J_2'x'^3, \qquad J_2' = \tfrac{1}{3}(1 - \tfrac{1}{2}\tan^2\delta)\sin^2 1'',$$
$$+ J_3'x'^3y', \qquad J_3' = (-\tfrac{5}{6}\tan\delta + \tfrac{1}{6}\tan^3\delta)\sin^3 1'',$$
$$+ J_4'x'y'^3, \qquad J_4' = -\tfrac{1}{3}\tan\delta \sin^3 1'',$$

[1] Tables for the Reduction of Astronomical Photographs. *Contrib. Col. Univ. Observatory*, 23.

$$
\begin{aligned}
y_c = y' &+ K_1' x'^2, & K_1' &= \tfrac{1}{2} \tan \delta \sin 1'', \\
&+ K_2' x' y', & K_2' &= \tfrac{1}{2} (1 - \tan^2 \delta) \sin^2 1'', \\
&+ K_3' y'^3, & K_3' &= \tfrac{1}{3} \sin^2 1'', \\
&+ K_4' x'^2 y', & K_4' &= -\tfrac{1}{2} \tan \delta \sin^3 1'', \\
&+ K_5' x'^4, & K_5' &= (\tfrac{5}{24} \tan \delta - \tfrac{1}{24} \tan^3 \delta) \sin^3 1''.
\end{aligned}
$$

The quantities herein indicated by x' and y' have the following value:

$$x' = (\alpha' - \alpha) \cos \delta, \quad y' = \delta' - \delta$$

where α and δ are the right-ascension and declination of the central star and α' and δ' are the right-ascension and declination of the star in question. The differences $(\alpha' - \alpha)$ and $(\delta' - \delta)$ are expressed in seconds of arc, so that x' and y' and consequently x_c and y_c are also in seconds of arc. In these computations no attention was paid to proper motion, the epochs of the plates and the *Astronomische Gesellschaft* Catalog being nearly the same.

A solution of the sixty equations due to Set (3) and of the fifty due to Set (4) gave the plate constants and residuals contained in Table VIII. The probable errors of one equation, and the residuals are much larger than in the foregoing solutions due to the fact that they now include the rather large errors of the meridian circle places.

With the values of p, r, k and c of Table VIII the mean coördinates given in Tables V and VI were next corrected, giving the standard coördinates for Sets (3) and (4) designated by X_3, Y_3 and X_4, Y_4 in Table IX. For stars appearing in both sets these coördinates were now averaged after weights were given to the coördinates according to the following rule: Stars appearing on all four plates of either set are given a weight of one for that set, stars appearing on two or three plates of a set are given a weight of three fourths for that set, and if appearing on only one of the four plates are given a weight of one half.

Intercomparison of Sets (3) and (4).— The coördinates of the stars for which it was possible to obtain an average were now taken to be the Final Coördinates for Series (B) designated by X_B and Y_B, and the standard coördinates of Set (3) were com-

TABLE VIII. — COMPARISON OF SET (3) AND SET (4) WITH ASTRONOMISCHE GESELLSCHAFT CATALOG.

Set (3).			Set (4).		
$p = +0.000448 \pm 0.000044$			$p = +0.000311 \pm 0.000046$		
$r = -0.000584 \pm 0.000044$			$r = +0.000194 \pm 0.000046$		
$k = +2''.487 \pm 0''.102$			$k = +1''.814 \pm 0''.103$		
$c = +0''.105 \pm 0''.102$			$c = -0''.219 \pm 0''.103$		
Prob. error of one equ. $= \pm 0''.5536$			Prob. error of one equ. $= \pm 0''.5159$		
Star.	v_x	v_y	Star.	v_x	v_y
	″	″		″	″
2	—1.67	+1.39	2	—1.42	+1.15
3	—0.67	—0.47	3	—0.79	—0.90
6	+1.18	+0.42	6	+1.43	+0.35
8	—0.10	—0.16	8	+0.01	—0.04
9	—0.65	+0.59	9	—0.58	+0.60
11	+1.97	+0.12	11	+2.01	+0.21
12	+0.42	—0.65	12	+0.39	—0.67
14	—1.07	—0.02	14	—0.98	+0.12
15	+0.07	—0.57	15	—0.45	—0.35
16	—0.20	+0.35	16	—0.30	+0.59
17	—0.06	+0.43	17	—0.17	+0.34
19	+1.80	—0.32	19	+0.59	—0.50
20	+2.49	+0.10	20	+1.81	—0.22
21	+0.35	—0.39	21		
22	—0.63	—0.60	22	—1.08	—0.70
23	—0.81	—0.46	23	—0.83	—0.23
24	—1.59	—0.16	24		
25	+0.15	+0.31	25	+0.02	+0.12
26	—0.47	—0.29	26	—0.76	—0.27
27	+1.17	—0.10	27	+1.02	—0.19
30	+0.25	+0.15	30	+0.14	+0.30
31	—0.95	—1.17	31	—1.33	—0.70
35	+0.72	+0.10	35	+0.60	+0.18
36	+0.56	—0.35	36	+0.51	—0.41
37	—0.50	+0.33	37	—0.46	+0.17
38	+0.83	+0.43	38	+0.82	+0.51
39½	—0.48	+0.55	39½		
40	—1.36	+0.41	40		
41½	—0.73	—0.39	41½		
42	0.00	+0.43	42	—0.20	+0.53

TABLE IX. — FINAL COÖRDINATES OF SERIES (B).

Star.	X_3	X_4	X_B	Y_3	Y_4	Y_B
2	—2930.10	—2929.86	—2929.98	+ 3.11	+ 2.87	+ 2.99
3	—2300.92	—2301.05	—2300.98	+1655.62	+1655.20	+1655.41
5	—2059.95	—2059.49	—2059.77	+2873.98	+2774.06	+2874.01
6	—2000.92	—2000.67	—2000.80	— 127.20	— 127.26	— 127.23
8	—1804.62	—1804.52	—1804.57	—2134.90	—2134.77	—2134.84
9	—1707.44	—1707.38	—1707.41	+1899.63	+1899.64	+1899.64
11	—1313.59	—1313.55	—1313.57	— 110.48	— 110.39	— 110.44
12	—1120.43	—1120.46	—1120.44	—1297.69	—1297.70	—1297.70
13	—1097.59	—1097.66	—1097.62	+ 381.59	+ 381.30	+ 381.47
14	— 831.91	— 831.83	— 831.87	—2151.94	—2151.80	—2151.87
15	— 618.43	— 618.96	— 618.66	+2744.20	+2744.41	+2744.29
16	— 199.93	— 200.03	— 199.98	—2604.84	—2604.61	—2604.72
17	— 137.90	— 138.01	— 137.96	+1138.17	+1138.08	+1138.12
18	— 98.66	— 98.78	— 98.72	+1348.32	+1348.38	+1348.35
19	— 25.55	— 26.77	— 26.16	— 19.73	— 19.90	— 19.82
20	+ 2.49	+ 1.81	+ 2.15	+ 0.10	— 0.22	— 0.06
21	+ 76.71	+ 76.50	+ 76.64	—1337.01	—1336.77	—1336.93
22	+ 90.39	+ 89.94	+ 90.20	+2070.68	+2070.57	+2070.63
23	+ 306.13	+ 306.10	+ 306.12	—2264.63	—2264.40	—2264.52
24	+ 350.53	+ 350.88	+ 350.67	— 53.01	— 52.66	— 52.87
25	+ 412.41	+ 412.30	+ 412.36	+1845.76	+1845.59	+1845.68
26	+ 619.36	+ 619.26	+ 619.31	+2698.46	+2698.48	+2698.47
27	+1078.59	+1078.44	+1078.52	+1073.80	+1073.72	+1073.76
28	+1078.89		+1078.84	+1079.37		+1079.38
30	+1307.09	+1306.98	+1307.04	— 168.49	— 168.33	— 168.41
31	+1377.12	+1376.75	+1376.94	—1964.07	—1963.60	—1963.84
32	+1868.76		+1868.71	—1172.93		—1172.86
35	+1925.26	+1925.12	+1925.19	—2095.67	—2095.60	—2095.64
36	+1924.07	+1924.01	+1924.04	+1992.12	+1992.07	+1992.10
37	+2002.20	+2002.23	+2002.22	—1530.28	—1530.46	—1530.37
38	+2029.67	+2029.66	+2029.66	— 832.39	— 832.31	— 832.35
39	+2223.91		+2223.82	+1844.19		+1844.19
39½	+2296.67		+2296.59	+1398.00		+1398.01
40	+2536.24		+2536.15	+1135.33		+1135.34
41	+2670.11		+2670.02	+ 758.35		+ 758.37
41½	+2731.10		+2731.00	+2396.58		+2396.56
42	+2741.13	+2740.92	+2741.03	+1597.09	+1597.21	+1597.15

pared with these by determining the usual constants p, r, k and c from fifty-six[1] equations of the usual form.

These constants had the following values :

$$p = -0.0000230 \pm 0.0000048$$
$$r = -0.0000089 \pm 0.0000048$$
$$k = -0''.0234 \pm 0''.0109$$
$$c = +0''.0200 \pm 0''.0109$$

Probable error of one equation $= \pm 0''.0576$.

The corrections due to these constants were applied to the remaining standard coördinates of Set (3) thereby bringing about their transformation into the Final Coördinates of Series (B).

The averaged and computed Final Coördinates of Series (B) are also given in Table IX. These are now the photographic coördinates derived from the plates of Series (B), fully corrected, and brought into the best possible general agreement with the *Astronomische Gesellschaft* Catalog.

Preliminary Catalog. — The right ascensions and declinations corresponding to the Final Coördinates of Series (B) were now computed. The formulæ used were those of Contrib. 23, p. 6, and are given below, fourth and higher order terms were again omitted in the calculations.

$$\begin{aligned}
x' = x &+ J_1 xy, & J_1 &= \tan\delta \sin 1'',\\
&+ J_2 xy^2, & J_2 &= \tan^2\delta \sin^2 1'',\\
&+ J_3 x^3, & J_3 &= -\tfrac{1}{3}(1 + \tan^2\delta)\sin^2 1'',\\
&+ J_4 x^3 y, & J_4 &= -(\tan\delta + \tan^3\delta)\sin^3 1'',\\
&+ J_5 xy^3, & J_5 &= \tan^3\delta \sin^3 1'',\\
&+ J_6 x^3 y^2, & J_6 &= -2(\tan^2\delta + \tan^4\delta)\sin^4 1'',\\
&+ J_7 x^5, & J_7 &= (\tfrac{1}{5} + \tfrac{2}{5}\tan^2\delta + \tfrac{1}{5}\tan^4\delta)\sin^4 1'',\\
&+ J_8 xy^4. & J_8 &= \tan^4\delta \sin^4 1''.
\end{aligned}$$

$$\begin{aligned}
y' = y &+ K_1 x^2. & K_1 &= -\tfrac{1}{2}\tan\delta \sin 1'',\\
&+ K_2 x^2 y, & K_2 &= -\tfrac{1}{2}(1 + \tan^2\delta)\sin^2 1'',\\
&+ K_3 y^3, & K_3 &= -\tfrac{1}{3}\sin^2 1'',\\
&+ K_4 x^2 y^2, & K_4 &= -\tfrac{1}{2}\tan^3\delta \sin^3 1'',\\
&+ K_5 x^4, & K_5 &= (\tfrac{3}{8}\tan\delta + \tfrac{1}{8}\tan^3\delta)\sin^3 1'',\\
&+ K_6 x^4 y, & K_6 &= (\tfrac{3}{8} + \tfrac{3}{4}\tan^2\delta + \tfrac{3}{8}\tan^4\delta)\sin^4 1'',\\
&+ K_7 x^2 y^3, & K_7 &= \tfrac{1}{2}(1 - \tan^4\delta)\sin^4 1''.
\end{aligned}$$

In these x' and y', as before, have the following significance :

$$x' = (a' - a)\cos\delta, \quad y' = \delta' - \delta.$$

[1] The four equations given by β and β_2 were not used in this solution in order to prevent their large $(X_3 - X_B)$ from having an undue influence on the constants.

The right ascensions and declinations thus obtained are put down in Table X together with the difference between these and the positions given in the first paper on β Cygni[1] taken in the sense new *minus* old.

It will be seen that these check even now very closely, the variation from the mean differences in right ascension and declination exceeding 0″.50 in only six cases and 0″.63 in only one case.

Mean Rectangular Coördinates from Set (1) *and Set* (2). — On pages 341–346 inclusive of the paper just referred to are given the corrected distances and position angles of the stars of Series (A) with reference to β Cygni as center. Averages of these distances and angles were formed for Set (1) and Set (2) and the distances then reduced to seconds of arc by multiplying by 28.0124, which is the average number of seconds per division of the glass scale used in these measures. From the average distances and position angles rectangular coördinates were now computed. These were now referred to β_2 as center, in order to agree with the previous part of the work, and are so given in Table XI.

Standard Coördinates of Series (*A*). — The mean coördinates for Sets (1) and (2) as given in Table XI were now compared with the Final Coördinates of Series (B) after these also had been referred to β_2 as center. The comparison equations were formed in the usual way and the constants deduced for Set (1) and Set (2) as well as the residuals for the sixty-eight equations of Set (1) and sixty-six equations of Set (2) are given in Table XII. It will be seen that the probable errors of the single equations as well as the residuals themselves are very much smaller than those given in Table VIII, showing, as was to be expected, that the accuracy of the Preliminary Catalog is far greater than that of the *Astronomische Gesellschaft*, at least with respect to the differential coördinates.

The constants just obtained were used to correct the mean coördinates given in Table XI, thus forming the Standard Coör-

[1] The Rutherfurd Photographic Measures of the Stars about β Cygni. By H. Jacoby. *Annals N. Y. Acad Sci.*, Vol. VI.

TABLE X. — PRELIMINARY CATALOG AND COMPARISON WITH CATALOG IN FIRST PAPER ON β CYGNI.

Star.	Right Ascension 1875	Difference New — Old.	Declination 1875.	Difference New — Old.
	° ′ ″	″	° ′ ″	″
2	290 30 34.42	+1.16	27 42 5.38	—0.64
3	42 13.72	+1.29	28 9 41.81	—0.61
5	46 40.00	+0.48	28 30 1.55	0.00
6	48 4.46	+1.48	27 40 0.98	—0.47
8	51 56.35	+1.41	27 6 34.50	—0.38
9	53 25.71	+1.14	28 13 49.10	—0.59
11	291 1 0.29	+1.51	27 40 20.66	—0.46
12	4 42.16	+1.68	27 20 34.04	0.27
13	5 2.57	+1.32	27 48 33.23	—0.64
14	10 9.04	+1.30	27 6 20.65	—0.18
15	13 59.82	+0.95	28 27 56.93	—0.56
16	21 59.12	+1.22	26 58 48.67	—0.43
17	23 7.23	+0.94	28 1 11.38	—0.82
18	23 51.61	+1.25	28 4 41.62	—0.70
19	25 13.95	+1.29	27 41 53.48	—0.45
20	25 45.93	+1.33	27 42 13.24	—0.57
21	27 9.77	+0.70	27 19 56.38	—0.24
22	27 25.92	+1.01	28 16 43.85	—0.92
23	31 27.27	+1.12	27 4 28.75	—0.22
24	32 19.52	+0.02	27 41 20.27	—0.68
25	33 31.45	+1.20	28 12 58.71	—0.72
26	37 27.83	+0.86	28 27 11.11	—0.52
27	46 4.99	+0.81	28 0 5.55	—0.63
28	46 5.37	+0.97	28 0 11.17	—0.33
30	50 19.11	+0.55	27 39 22.72	—0.37
31	51 30.96	+0.81	27 9 27.16	—0.26
32	292 0 47.92	+1.56	27 22 36.07	+0.26
35	1 46.34	+0.85	27 7 13.14	—0.58
36	2 7.65	+0.84	28 15 20.52	—0.53
37	3 16.10	+1.66	27 16 37.85	—0.43
38	3 51.02	+0.88	27 28 15.77	—0.41
39	7 46.98	+1.05	28 12 51.01	+0.10
39½	9 6.58		28 5 24.46	
40	13 36.14	+0.80	28 1 0.32	—0.48
41	16 4.85	+0.51	27 54 42.51	—0.13
41½	17 26.81		28 21 59.98	
42	17 31.85	+1.25	28 8 40.68	—0.20

TABLE XI.—CORRECTED RECTANGULAR COÖRDINATES OF SET (1) AND SET (2).

Star.	x_1	y_1	x_2	y_2
1			—3041.48	+ 749.42
2	—2932.18	+ 2.95	—2931.34	+ 1.90
3	—2302.96	+1655.53	—2303.04	+1655.22
4			—2237.90	+2626.29
5	—2061.37	+2873.27	—2061.08	+2872.99
6	—2003.27	— 127.45	—2002.78	— 127.37
7	—1927.44	—1566.48	—1927.04	—1566.54
8	—1806.83	—2135.02	—1806.26	—2134.92
9	—1709.39	+1899.47	—1709.46	+1899.57
10	—1576.47	+3150.90	—1575.33	+3151.51
11	—1315.97	— 110.64	—1315.76	— 110.52
12	—1123.07	—1298.17	—1122.52	—1297.81
13	—1100.15	+ 381.33	—1099.40	+ 381.71
14	— 834.17	—2152.28	— 833.23	—2152.01
15	— 620.30	+2744.45	— 620.96	+2744.49
16	— 202.26	—2604.99	— 201.44	—2604.36
17	— 139.92	+1138.18	— 139.75	+1138.63
18	— 100.91	+1348.36	— 100.87	+1348.67
19	— 28.12	— 19.94	— 28.44	— 19.82
20	0.00	0.00	0.00	0.00
21	+ 74.21	—1337.44	+ 75.35	—1337.06
22	+ 88.20	+2070.74	+ 88.17	+2071.21
23	+ 303.92	—2265.02	+ 304.64	—2264.38
24			+ 349.52	— 52.62
25	+ 410.15	+1845.68	+ 410.23	+1846.04
26	+ 617.34	+2698.18	+ 617.36	+2698.52
27	+1076.72	+1073.61	+1076.75	+1074.24
28	+1077.01	+1078.94	+1076.86	+1079.80
29	+1253.96	—1421.58		
30	+1305.27	— 168.88	+1305.88	— 168.04
31	+1375.02	—1964.04	+1375.84	—1963.73
32	+1866.61	—1173.52		
33	+1871.09	+1968.94	+1870.50	+1969.98
34	+1895.67	+3140.42	+1895.67	+3141.17
35	+1922.94	—2095.75	+1924.03	—2094.67
36	+1922.13	+1991.69	+1922.02	+1992.57
37	+1999.93	—1530.40		
38	+2027.64	— 832.69	+2028.13	— 831.83
39	+2221.70	+1843.20	+2221.71	+1844.37
40	+2534.11	+1134.71	+2534.38	+1136.10
41	+2668.11	+ 757.23	+2668.64	+ 759.01
42	+2738.77	+1596.33	+2738.52	+1597.47

dinates for Sets (1) and (2) designated by X_1, Y_1 and X_2, Y_2 in Table XIII.

Intercomparison of Set (1) *and Set* (2). — This intercomparison was made in an exactly similar manner to that of Set (3) and Set (4). From the sixty-four equations of condition solved by the method of least squares the following constants were deduced:

$$p = -0.00000125 \pm 0.00000730$$
$$r = +0.00000064 \pm 0.00000730$$
$$k = -0''.0085 \pm 0''.0166$$
$$c = +0''.0077 \pm 0''.0166$$
Probable error of one equation $= \pm 0''.0927$

As was to be expected these quantities are all extremely small and the corrections due to p and r are entirely inappreciable. The corrections for k and c were applied with the sign given to the stars of Set (1) not appearing in (2), and with the opposite sign to stars that appeared in Set (2) but not in (1). The Final Coördinates of Series (A) are given in columns X_A and Y_A of Table XIII.

Final Coördinates. — The Final Cöordinate of Series (A) and (B) were now averaged for all stars appearing in both series and the Final Coördinates of Series (A) were compared with these by a least square solution embodying seventy equations of condition. The constants deduced by this solution were as follows:

$$p = +0.00000127 \pm 0.00000643$$
$$r = +0.00000035 \pm 0.00000643$$
$$k = -0''.0053 \pm 0''.0145$$
$$c = -0''.0025 \pm 0''.0145$$
Probable error of one equation $= \pm 0''.0846$

the total correction due to these constants never amounting to more than $0''.01$.

The averaged coördinates of Series (A) and (B) together with the corrected coördinates of those stars appearing in only one of the series are given in Table XIV as the Final Coördinates of the stars under consideration.

Magnitudes. — In order to determine the stellar photographic magnitudes the diameters of all the star-images were carefully measured on Plates 5 and 8, six settings being made for each

TABLE XII. — COMPARISON OF SET (1) AND SET (2) WITH PRELIMINARY CATALOG.

Set (1).			Set (2).		
$p = -0.0000160 \pm 0.0000128$			$p = -0.0000038 \pm 0.0000168$		
$r = -0.0000868 \pm 0.0000128$			$r = +0.0001684 \pm 0.0000168$		
$k = -0''.0203 \pm 0''.0292$			$k = -0''.4344 \pm 0''.0378$		
$c = +0''.3415 \pm 0''.0292$			$c = -0''.0746 \pm 0''.0378$		
Prob. error of one equ. $= \pm 0''.1688$			Prob. error of one equ. $= \pm 0''.2141$		
Star.	v_x	v_y	Star.	v_x	v_y
	″	″		″	″
2	−0.02	−0.01	2	+0.37	−0.71
3	+0.04	+0.18	3	−0.05	+0.07
5	+0.31	−0.68	5	+0.91	−0.81
6	−0.30	−0.11	6	−0.28	−0.07
8	+0.08	−0.02	8	−0.34	+0.11
9	+0.01	−0.07	9	0.00	+0.09
11	−0.24	−0.03	11	−0.49	+0.02
12	−0.37	−0.26	12	−0.59	−0.04
13	−0.42	+0.04	13	0.00	−0.30
14	+0.03	−0.17	14	−0.02	−0.12
15	+0.26	+0.34	15	−0.10	+0.16
16	+0.08	+0.04	16	−0.20	+0.27
17	+0.07	+0.31	17	+0.12	+0.40
18	−0.18	+0.26	18	−0.20	+0.20
19	+0.17	+0.16	19	−0.57	−0.13
20	−0.02	+0.34	20	−0.43	−0.08
21	−0.18	−0.20	21	+0.19	−0.27
22	−0.05	+0.37	22	+0.05	+0.42
23	+0.12	−0.16	23	−0.16	−0.04
24			24	+0.56	+0.05
25	−0.25	+0.29	25	−0.09	+0.15
26	−0.08	0.00	26	+0.23	−0.20
27	+0.22	+0.21	27	+0.13	+0.15
28	+0.19	−0.08	28	−0.08	+0.09
30	+0.35	−0.07	30	+0.52	+0.01
31	+0.36	+0.23	31	+0.27	−0.26
32	+0.10	−0.20	32		
35	+0.03	+0.37	25	+0.18	+0.51
36	+0.02	+0.01	36	+0.04	−0.01
37	−0.06	+0.45	37		
38	+0.15	+0.13	38	+0.03	+0.04
39	−0.19	−0.54	39	−0.08	−0.35
40	−0.05	−0.15	40	+0.13	+0.18
41	+0.11	−0.64	41	+0.46	+0.04
42	−0.31	−0.33	42	−0.53	−0.30

TABLE XIII. — FINAL COÖRDINATES OF SERIES (*A*).

Star.	X_1	X_2	X_A	Y_1	Y_2	Y_A
1		−3041.77	−3041.76		+ 749.88	+ 749.87
2	−2932.15	−2931.67	−2931.96	+ 3.04	+ 2.34	+ 2.69
3	−2303.08	−2303.17	−2303.13	+1655.64	+1655.54	+1655.59
4		−2237.86	−2237.85		+2626.60	+2626.59
5	−2061.61	−2061.16	−2061.38	+2873.38	+2873.27	+2873.32
6	−2003.25	−2003.22	−2003.24	− 127.28	− 127.09	− 127.19
7	−1927.29	−1927.63	−1927.46	−1566.27	−1566.26	−1566.26
8	−1806.63	−1807.05	−1806.84	−2134.81	−2134.66	−2134.74
9	−1709.54	−1709.55	−1709.54	+1899.63	+1899.79	+1899.71
10	−1576.74	−1575.10	−1575.92	+3151.05	+3151.70	+3151.38
11	−1315.96	−1316.20	−1316.08	− 110.42	− 110.36	− 110.39
12	−1122.96	−1123.18	−1123.07	−1297.91	− 1297.65	−1297.78
13	−1100.18	−1099.76	−1099.97	+ 381.56	+ 381.83	+ 381.69
14	− 833.99	− 834.03	− 834.01	−2151.98	−2151.93	−2151.96
15	− 620.55	− 620.91	− 620.73	+2744.70	+2744.52	+2744.61
16	− 202.05	− 202.32	− 202.18	−2604.63	−2604.38	−2604.50
17	− 140.04	− 139.98	− 140.01	+1138.49	+1138.58	+1138.54
18	− 101.05	− 101.06	− 101.06	+1348.67	+1348.61	+1348.64
19	− 28.14	− 28.87	− 28.50	− 19.60	− 19.88	− 19.74
20	− 0.02	− 0.43	− 0.22	+ 0.34	− 0.07	+ 0.14
21	+ 74.31	+ 74.69	+ 74.50	−1337.07	−1337.13	−1337.10
22	+ 88.00	+ 88.10	+ 88.05	+2071.06	+2071.11	+2071.08
23	+ 304.10	+ 303.82	+ 303.96	−2264.61	−2264.49	−2264.55
24		+ 349.08	+ 349.09		− 52.75	− 52.76
25	+ 409.96	+ 410.12	+ 410.04	+1846.03	+1845.89	+1845.96
26	+ 617.08	+ 617.40	+ 617.24	+2698.53	+2698.33	+2698.43
27	+1076.59	+1076.51	+1076.55	+1074.02	+1073.98	+1074.00
28	+1076.88	+1076.62	+1076.75	+1079.35	+1079.54	+1079.44
29	+1254.04		+1254.03	−1421.12		−1421.11
30	+1305.25	+1305.41	+1305.33	− 168.43	− 168.34	− 168.38
31	+1375.15	+1375.06	+1375.10	−1963.55	−1964.03	−1963.79
32	+1866.66		+1866.65	−1173.00		−1172.99
33	+1870.87	+1870.40	+1870.64	+1969.41	+1969.57	+1969.49
34	+1895.35	+1895.78	+1895.56	+3140.87	+3140.76	+3140.82
35	+1923.07	+1923.23	+1923.15	−2095.21	−2095.06	−2095.14
36	+1921.91	+1921.93	+1921.92	+1992.17	+1992.16	+1992.16
37	+2000.01		+2000.00	−1529.87		−1529.86
38	+2027.66	+2027.55	+2027.60	− 832.16	− 832.25	− 832.20
39	+2221.48	+2221.59	+2221.54	+1843.70	+1843.90	+1843.80
40	+2533.95	+2534.14	+2534.04	+1135.25	+1135.59	+1135.42
41	+2667.98	+2668.33	+2668.16	+ 757.79	+ 758.48	+ 758.14
42	+2738.57	+2738.36	+2738.46	+1596.88	+1596.91	+1596.90

TABLE XIV.—FINAL COÖRDINATES.

Star.	X_f	Y_f	Star.	X_f	Y_f
1	—3041.77	+ 749.87	23	+ 303.96	—2264.50
2	—2932.04	+ 2.87	24	+ 348.80	— 52.78
3	—2303.13	+1655.53	25	+ 410.12	+1845.85
4	—2237.86	+2626.59	26	+ 617.20	+2698.48
5	—2061.65	+2873.70	27	+1076.46	+1073.91
6	—2003.10	— 127.18	28	+1076.72	+1079.44
7	—1927.47	—1566.26	29	+1254.03	—1421.11
8	—1806.78	—2134.76	30	+1305.11	— 168.36
9	—1709.55	+1899.70	31	+1374.94	—1963.78
10	—1575.93	+3151.38	32	+1866.60	—1172.90
11	—1315.90	— 110.38	33	+1870.64	+1969.49
12	—1122.83	—1297.71	34	+1895.56	+3140.82
13	—1099.87	+ 381.61	35	+1923.10	—2095.36
14	— 834.02	—2151.88	36	+1921.90	+1992.16
15	— 620.77	+2744.48	37	+2000.04	—1530.08
16	— 202.16	—2604.58	38	+2027.56	— 832.24
17	— 140.06	+1138.36	39	+2221.60	+1844.02
18	— 100.96	+1348.52	39½	+2294.44	+1398.07
19	— 28.40	— 19.75	40	+2534.02	+1135.41
20	— 0.11	+ 0.07	41	+2668.02	+ 758.28
21	+ 74.50	—1336.98	41½	+2728.85	+2396.62
22	+ 88.05	+2070.88	42	+2738.67	+1597.06

star on both the plates. The average results of these measures are set down in Table XV, in which the quantities are expressed in revolutions of the micrometer screw, equivalent to half-millimeters.

These diameters were to be compared with the *Astronomische Gesellschaft* Catalog [1] magnitudes by determining two constants a and b such that the photographic magnitude m can be computed from the diameters D of the star images by the formula

$$m = a - b \log D.$$

Before doing this, however, it was thought advisable to exclude such stars whose photographic magnitudes differed decidedly from the visual magnitudes. For this purpose a weighted mean value for a and b was determined from previous magnitude determinations of Rutherford plates, and with these values amounting to

$$a = +14.8, \qquad b = -5.4.$$

the magnitudes of the stars were calculated.

[1] For stars 1, 4, 13, 18 and 39, *B.D.* Mags. were used as these stars as well as 28, 32, 33, and 41, are not given in the *A.G.C.*

TABLE XV. — DIAMETERS OF STAR-IMAGES.

Star.	Diameter.	Star.	Diameter.	Star.	Diameter.
	r		r		r
1	0.082	16	0.155	31	0.137
2	0.114	17	0.140	32	0.136
3	0.126	18	0.124	33	0.114
4	0.120	19	0.396	34	0.143
5	0.104	20	0.338	35	0.156
6	0.167	21	0.123	36	0.148
7	0.130	22	0.144	37	0.136
8	0.168	23	0.168	38	0.152
9	0.138	24	0.109	39	0.112
10	0.117	25	0.133	39½	0.103
11	0.158	26	0.194	40	0.122
12	0.147	27	0.182	41	0.112
13	0.102	28	0.118	41½	0.100
14	0.168	29	0.134	42	0.186
15	0.112	30	0.128		

This preliminary solution showed that stars 8, 14, 15, 19 and 20, had best be excluded from the least square solution, for a and b, as their photographic magnitudes differed by more than 0.4 mag. from the visual.

Thirty-four equations of the form

$$a + \log(100D)\, b - m = 0$$

were now solved for a and b and the following values found:

$$a = +13.60 \pm 0.37, \qquad b = -4.24 \pm 0.35.$$

The photographic magnitudes given in the Final Catalog were computed with these constants. It will be noted that although the actual values of a and b as determined by the least square solution differ from the means above given, the sum $a + b$ agrees very closely in the two cases.

Final Catalog. — In Table XVI is given the Final Photographic Catalog. The right ascensions and declinations were computed by the same formulæ with which the preliminary places were determined, while the precessions and secular variations were calculated with the aid of Becker's "Tafeln zur Berechnung der Præcession," suitable corrections being applied to reduce the epoch to 1875.0. The numbers of the stars correspond to those given in Jacoby's discussion of the first six negatives. Stars $39\frac{1}{2}$ and $41\frac{1}{2}$ were not measured on these

plates and were inserted between the original numbers so as to avoid any changes.

Discussion of Results. — As is indicated by the probable errors of the single equations throughout the work, the micrometric positions of the surrounding stars with respect to β and β_2 (Nos. 19 and 20) are of a very much higher degree of accuracy than the meridian circle places of the *Astronomische Gesellschaft* Catalog.

The comparatively large difference between the position of No. 19 of this Catalog and No. 10049 *A.G.C.* is in all probability due to a systematic error of about $+ 0^s.08$ in right ascension and $- 0''.4$ in declination, in the *A.G.C.* places of the small stars of this region.

The right ascension of No. 10052 *A.G.C.* as given in this Catalog is in all probability a typographical error, and should be read $19^h\ 25^m\ 42^s.97$.

From the standard coördinates of β_2, (No. 20) for the mean dates of Sets (3) and (4) as given in Table IX, and for the mean dates of Sets (1) and (2) as given in Table XIII, it was not possible to come to any definite conclusion as to the true nature of the photographic displacement of this star. But we are certain that the photographic displacement does exist and if not due to parallax, or a large difference in the relative distances from us, of β and the neighboring stars, it may be due to the difference in the magnitude or spectra of the double star β and the neighboring small stars.

In order to make a thorough enquiry into the nature of the photographic displacement of β Cygni we should have a series of plates taken, at intervals of a few days, during that period of one year in which it may be photographed. As soon as the possibility of obtaining such a series presents itself the work on this subject will be continued.

In conclusion I wish to express my thanks to Professor Jacoby for many helpful suggestions and for securing the publication of the work.

TABLE XVI. — FINAL CATALOG.

No.	Magnitude. Vis.	Photo.	No. of Plates.	Right Ascension 1875.0 In Time.	In Arc.	Precession. In Time.	In Arc.	Secular Variation. In Time.	In Arc.	Declination 1875.0	Precession.	Secular Variation.	Mean Epoch.
				h m s	° ′ ″	s	″	s	″	° ′ ″	″	″	
1	9.3*	9.7	2	19 21 53.603	290 28 24.05	+2.40883	+ 36.1325	+0.00107	+ 0.0161	+27 54 31.23	+7.0144	+0.3262	1875.72
2	9.1	9.1	11	22 2.301	30 34.52	.41478	36.2217	.00106	.0159	27 42 5.18	.0263	.3269	1876.11
3	9.0	8.9	14	22 48.915	42 13.73	.40276	36.0414	.00108	.0162	28 9 39.85	.0900	.3248	1876.20
4	9.2*	9.0	1	22 53.429	43 21.43	.39523	35.9284	.00108	.0162	28 25 53.12	.0961	.3238	1875.72
5	9.0	9.3	8	23 6.688	46 40.31	.39354	35.9031	.00110	.0165	28 30 1.16	.1142	.3234	1876.00
6	8.2	8.4	14	23 12.286	48 4.29	+ 2.41700	+ 36.2550	+0.00105	+0.0157	27 40 0.96	+7.1218	+ 0.3265	1876.20
7	9.2	8.9	4	23 18.509	49 37.62	.42818	36.4227	.00104	.0156	27 16 2.37	.1303	.3280	1875.68
8	9.0	8.4	14	23 27.752	51 56.29	.43269	36.4904	.00102	.0153	27 6 34.50	.1429	.3285	1876.20
9	8.7	8.8	14	23 33.715	53 25.72	.40165	36.0248	.00108	.0162	28 13 49.08	.1510	.3242	1876.20
10	8.8	9.1	2	23 43.440	55 51.59	.39203	35.8804	.00110	.0165	28 34 41.10	.1643	.3228	1875.64
11	8.0	8.5	14	24 4.006	291 1 0.09	+2.41779	+36.2668	+ 0.00106	+0.0159	27 40 20.66	+ 7.1923	+0.3201	1876.20
12	9.0	8.7	14	24 18.793	4 41.90	.42718	36.4077	.00104	.0156	27 20 33.96	.2124	.3272	1876.20
13	9.3*	9.3	12	24 20.164	5 2.46	.41428	36.2142	.00106	.0159	27 48 33.30	.2143	.3255	1876.10
14	9.0	8.4	13	24 40.603	10 9.04	.43410	36.5115	.00103	.0154	27 6 20.57	.2420	.3280	1876.24
15	8.0	9.2	11	24 55.991	13 59.87	.39658	35.9487	.00109	.0164	28 27 57.06	.2630	.3228	1876.25
16	8.3	8.6	14	25 27.939	21 59.09	+2.43839	+36.5758	+0.00102	+0.0153	26 58 48.75	+7.3064	+0.3281	1876.20
17	8.9	8.7	14	25 32.486	23 7.28	.40976	36.1464	.00107	.0160	28 1 11.56	.3126	.3242	1876.20
18	9.1*	9.0	14	25 35.434	23 51.51	.40819	36.1228	.00108	.0162	28 4 41.73	.3166	.3239	1876.20
19	3.0	6.8	14	25 40.924	25 13.85	.41886	36.2829	.00105	.0158	27 41 53.49	.3240	.3252	1876.20
20	6.7	7.1	14	25 43.054	25 45.81	.41875	36.2812	.00105	.0158	27 42 13.31	.3269	.3252	1876.20
21	8.9	9.0	9	25 48.652	27 9.78	+2.42911	+36 4366	+ 0.00104	+0.0156	27 19 56.27	+ 7.3345	+ 0.3266	1876.14
22	8.9	8.7	13	25 49.729	27 25.92	.40285	36.0428	.00109	.0164	28 16 44.04	.3359	.3230	1876.16
23	8.7	8.4	14	26 5.817	31 27.26	.43650	36.5475	.00103	.0154	27 4 28.71	.3577	.3274	1876.20
24	8.9	9.2	5	26 9.323	32 19.84	.41965	36.2948	.00106	.0159	27 41 20.30	.3625	.3251	1876.36
25	8.9	8.8	14	26 14.090	33 31.35	.40507	36.0760	.00108	.0162	28 12 58.82	.3690	.3231	1876.20

TABLE XVI. — FINAL CATALOG. — *Concluded.*

No.	Magnitude.		No. of Plates.	Right Ascension 1875.0		Precession.		Secular Variation.		Declination 1875.0	Precession.	Secular Variation.	Mean Epoch.
	Vis.	Photo.		In Time.	In Arc.	In Time.	In Arc.	In Time.	In Arc.				
				h m s	° ′ ″	s	″	s	″	° ′ ″	″	″	
26	8.0	8.1	14	19 26 29.859	291 37 27.88	+2.39874	+35.9811	+0.00109	+0.0164	+28 27 11.05	+7.3904	+0.3221	1876.20
27	8.2	8.3	14	27 4.339	46 5.09	.41202	36.1803	.00107	.0160	28 0 5.64	.4371	.3235	1876.20
28		9.1	7	27 4.360	46 5.41	.41198	36.1797	.00107	.0160	28 0 11.17	.4371	.3235	1876.07
29	8.8	8.8	1	27 17.146	49 17.20	.43140	36.4710	.00104	.0156	27 18 30.18	.4545	.3261	1875.57
30	8.8	8.9	14	27 21.291	50 19.36	.42191	36.3286	.00106	.0159	27 39 22.70	.4601	.3247	1876.20
31	8.8	8.8	13	27 26.076	51 31.13	+2.43570	+36.5355	+0.00103	+0.0154	+27 9 27.14	+7.4665	+0.3266	1876.24
32		8.8	4	28 3.198	292 0 47.97	.43040	36.5460	.00104	.0156	27 22 35.96	.5168	.3255	1876.20
33		9.1	2	28 4.623	1 9.35	.40628	36.0942	.00108	.0162	28 14 58.11	.5186	.3222	1875.64
34	9.1	8.7	5	28 6.940	1 44.11	.39724	35.9586	.00110	.0165	28 34 29.07	.5218	.3209	1875.66
35	8.4	8.5	14	28 7.094	1 46.41	.43748	36.5622	.00102	.0153	27 7 13.35	.5220	.3263	1876.20
36	8.9	8.6	13	28 8.512	2 7.67	+2.40619	+36.0928	+0.00108	+0.0162	+28 15 20.51	+7.5240	+0.3221	1876.16
37	9.0	8.8	9	28 13.072	3 16.07	.43330	36.4996	.00104	.0156	27 16 38.16	.5301	.3257	1876.27
38	8.3	8.6	14	28 15.405	3 51.08	.42803	36.4204	.00104	.0156	27 28 15.81	.5333	.3250	1876.20
39	9.4*	9.2	6	28 31.128	7 46.91	.40779	36.1168	.00108	.0162	28 12 50.77	.5544	.3221	1876.02
39½	9.5	9.3	3	28 36.439	9 6.58	.41134	36.1701	.00108	.0162	28 5 24.44	.5617	.3225	1876.41
40	9.0	9.0	5	28 54.411	13 36.16	+2.41373	+36.2060	+0.00106	+0.0159	+28 1 0.31	+7.5859	+0.3226	1876.11
41		9.2	5	29 4.334	16 5.02	.41683	36.2524	.00106	.0159	27 54 42.34	.5993	.3230	1876.11
41½	9.1	9.4	3	29 9.788	17 26.82	.40430	36.0645	.00108	.0162	28 21 59.96	.6067	.3212	1876.41
42	8.2	8.2	14	29 10.107	17 31.60	.41049	36.1574	.00107	.0160	28 8 40.51	.6071	.3220	1876.20

* *B. D.* Magnitude.

VITA.

Morris Francis Weinrich was born in Vienna, Austria, March 27, 1882. In 1885 his parents came to the United States, where he spent his childhood on a ranch in Irondale, Missouri, until 1889, when his parents moved their residence to St. Louis, Missouri, he here receiving his school education at the Smith Academy and the St. Louis Manual Training School, from which latter he graduated in 1899. Hereupon he entered Washington University where he remained one year. In 1900 he continued his studies at Columbia University, receiving the degree of Mech.E. in 1904. The remainder of this year he was in California and Nebraska in connection with the engineering department of the American Beet Sugar Company. During the year 1905 he was with the N. Y. and N. J. Telephone Company until February, 1906, when he was appointed assistant in the department of Engineering Drafting of Columbia University, which position he occupied until July, 1908. During this time he took graduate work under the Faculty of Pure Science, receiving the degree of A.M. in 1907. In April, 1908, he was appointed University Fellow in Astronomy for the year 1908–1909, which position he now holds.

MAKERS
SYRACUSE, - N.Y.
PAT. JAN. 21, 1908